METAIDENTIFICATION OF THE DAY-AHEAD MARKET SYSTEM OF THE POLISH ELECTRIC POWER EXCHANGE IN TERMS OF CONTROL AND SYSTEMS THEORY[1]

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Abstract: This paper contains the results of investigations concerning the metaidentification of the Day Ahead Market System as a Management System, which were obtained on the basis of a catalogue of 35 models of the Day Ahead Market System. In metaidentification, the input quantities are parameters of polynomials associated with the input quantities of the Day Ahead Market System, i.e. the volume of electricity supplied and sold at particular times of the day, and the output quantities are the parameters of the polynomial associated with the output quantities, i.e. with the weighted average electricity prices obtained in the result of the sale of electricity every hour of the day. As a result of metaidentification, a metamodel of the Day Ahead Market System is obtained, just like the identification results in Day Ahead Market System models. Discrete parametric models, parametric continuous models and continuous models in the state space were obtained.

Keywords: modelling, identification, multi-modelling, polynomials, meta-identification, Day Ahead Market

Selected abbreviations and symbols:

DAM system – Day-Ahead system,

identification – identification to obtain the model of the DAM system
(the object of identification is the system),

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meta-identification – meta-identification to obtain a DAM system meta-model (the object of identification is the model),

TGE S.A. – Towarowa Giełda Energii S.A. (Polish Power Exchange S.A.),

ee – electrical energy,

arx model – parametric model of the arx type (e.g. arx441),

z^{-i} – a time shift operator that performs a time delay of the signal by a number of units i,

PEPE system – The Polish Electricity Power Exchange system,

MISO models – Multi Input Singe Output models,

ss model – state space model,

A – system (process) matrix, i.e. the degree of internal organization of the DAM system,

B – input matrix, i.e. the control level of the DAM system,

C – output matrix, i.e. measurement system responses,

D – transmission matrix of the DAM system,

y_i – output from the DAM system (volume-weighted average price [PLN/MWh]),

u_j – input to the DAM system (volume of ee [MWh]),

y_{im} – output from DAM model (factor of the element of the polynomial $A_i(z)$),

u_{jm} – input to DAM model (factor of the element of the polynomial $B_j(z)$),

x_i – i-th state variable in the DAM model,

x_{im} – i-th state variable in the DAM meta-model.

1. Introduction

In order to obtain a model of a system, its modeling is carried out, which may be analytical, identifying (input-output, "econometric"), or, recently, neural (see: Bubnicki, 1974; Tchórzewski, 2013). Modeling leads, in effect, to a system model (Zimmer and Englot, 2005). In the experiment under consideration here, the system that we address is the Day-Ahead Market (DAM) system operating at the Polish Energy Exchange, i.e. Towarowa Giełda Energii S.A. (TGE S.A.), regarding the conclusion of transactions for the sale of electrical energy (ee), which is called for the purpose of this paper the Polish Electricity Power Exchange (PEPE) (see: Towarowa Giełda Energii S.A., 2020).

In the case of identification modeling (in short: identification), in order to obtain a system model, it is necessary to have at disposal appropriate numerical data about the system, preferably measured at equal intervals of time, from a sufficiently long period of its operation.

The obtained data may be of different nature. In the here analyzed experiment the data were collected from the DAM on the PEPE in the form of the volume of electricity supplied and sold in each hour of the day and the volume-weighted average price obtained for electricity sold in each hour of the day, recorded in the period between January 1st, 2013 and April 31st, 2016 (see: Towarowa Giełda Energii S.A., 2020).

The model structure was selected for the conditions of the experiment, which was the arx model, this model form being, for the recorded data, the closest to them. Then, it was decided that if we want to obtain a management system model, it is necessary to define which quantities should be the inputs to the Day-Ahead Management System management system, and which quantities should be the outputs from the Day-Ahead Management System.

In terms of control theory and systems, the Day-Ahead system consists of two subsystems, i.e. the Day-Ahead Management System and the Day-Ahead Market Executive System (see: Kaczorek et al., 2014; Konieczny, 1983; Tchórzewski, 2013). The experiment carried out in this study covers the Day-Ahead Management System as a subsystem of the Day-Ahead Market.

This is one side of the experiment. The other side of the experiment concerns the identification carried out for the parameters of models obtained for the same length of data periods moving in time with a fixed step of change (here one month) in order to obtain a model of the obtained system models, i.e. a metamodel. The process of modeling and meta-modeling of the system managed by DAM, leading to obtaining models and meta-model of the system is shown in Fig. 1.

In order to unequivocally present the research problem, an attempt was made to define the basic concepts used in the studied experiment, including the following basic definitions:

Definition 1. The DAM system - a subsystem in the DAM system that processes the information on the volume of ee delivered to and quoted on the DAM (input quantity), leading to a decision on the amount of the received average price weighted by the volume of delivered and sold ee, quoted on the DAM at each hour of the day (output value) – see Fig. 2.

Definition 2. The DAM system model – a substitute scheme of the DAM system obtained as a result of identification of the DAM system, in the form of the arx parametric model based on the recorded values of the input quantities of the volume of ee type in each hour of the day [MWh] and the output quantities of the average price weighted by the volume of ee received every hour of the day [PLN/MWh].

Figure 1. Graphical illustration of the modeling and metamodeling process of the Day-Ahead Market System (The DAM System) on the TGE S.A.
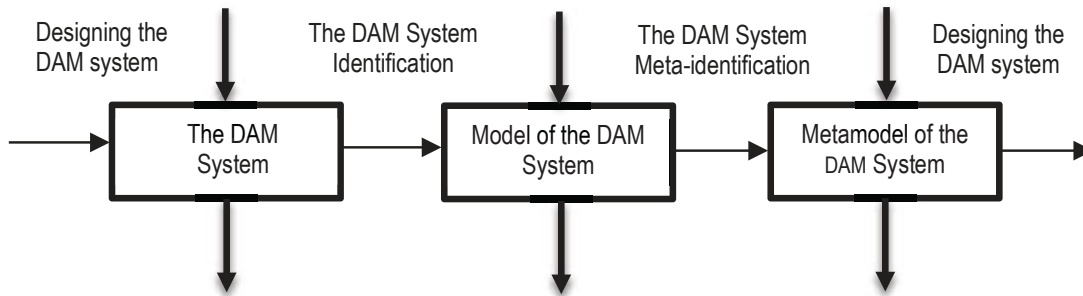
Source: own study.

Definition 3. The metamodel of the DAM system - a substitute diagram of the DAM system model, obtained as a result of meta-identification in the form of a parametric metamodel arx obtained on the basis of the values of the $B_j(z)$ polynomial coefficients for each hour of the day, being the result of identification of the rolling values of the input quantities and the result of identifying the values of the output quantities, such as the values of the coefficients of the $A_i(z)$ polynomial for each hour of the day in the rolling model with the hourly step, where $A_i(z)$ and $B_j(z)$, appearing in the DAM model of the arx type are the identification results, expressed in the form of appropriate polynomials with a delay parameter indicating how many time units a given part of the polynomial is delayed in relation to the current time of the input or output signal (e.g. $y(t) z^{-2} = y(t-2)$).

Definition 4. The implementation (executive) system of the DAM management system is a subsystem that performs tasks related to the supply of electricity on the DAM on PEPE and sale on the DAM. The results of identification and meta-identification are stored in the **theta** format matrix (**th** matrix), which contains the entire information, concerning the model or metamodel, its structure and parameter estimators, along with their estimation by covariance, where the **th** matrix has the theta format, which is used in the MATLAB environment and is the system model identification result matrix (**theta** matrix). It contains all the information regarding the model, its structure and parameter estimators along with their estimation by covariance.

The respective **theta** matrix has the strictly defined dimensions. Each element of the theta matrix contains specific information. And so, the elements of the first line contain: variance parameter estimators, sampling interval T and parameters *na*, *nb*, *nk*, etc., explained below. The second line includes: the Final Prediction Error indicator (FPE indicator), year, month, day, minute, and the numerical code of the command, according to which the model was generated. Then, the third row shows the estimators of the model parameters in alphabetical order: *a*1, *a*2, ..., *b*0, *b*1, ... (zeros and ones at the beginning of the polynomial are omitted). Finally, the lines from 4 to $(3 + n^2)$ contain the estimate of the covariance matrix, while *na* specifies the

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[2] *n* – row number in the **th** matrix.

number of poles (roots of the denominator), (nb-1) - specifies the number of zeros (roots of the numerator), and nk^3 - the number of factors related to the delay of the signal y in relation to the signal u (see: Tchórzewski, 2013, Zimmer and Englot, 2005).

The process of finding a representation of the matrix of measurement data in the form of a formalized identification or meta-identification of the DAM system is represented as:

$$Z^N = [\text{output, input}] \tag{1}$$

in a vector of model parameters [θ] stored in the form of the **th** matrix of **theta** format, where N is the number of ordered observations of successive values of the input variables and successive values of the output variables.

2. Characteristics of the Day-Ahead Market System on the Polish Electricity Power Exchange System

The Polish Electricity Power Exchange (PEPE) System is becoming an increasingly important subsystem of the Polish Electricity Market (PEM) system every year. Transactions on the PEPE start operating on the Day-Ahead Market (DAM) on the day preceding the physical supply of electrical energy (ee), then their activity is being extended to longer periods, from a week to a month, and even sometimes up to a year.

Due to the tendency to shorten the preparation time of generating units' work schedules, the Day-Ahead Electricity Market of the "day before" type changes to the "hour before" type market. This involves the possibility of submitting offers several times a day (see Mielczarski, 2000; Tchórzewski and Marlęga, 2020; TGE S.A., 2020, and also Sprawozdanie URE, 2017, 2020; Raport URE, 2014).

Transactions on the Day-Ahead Market in terms of the contract form are placed between long-term bilateral contracts and transactions on the Balancing Market. The Day-Ahead Market in Poland has been operating since June 30, 2000, as a spot market for electricity. From the beginning of trading, prices on the DAM were a reference for the electricity prices in bilateral contracts. The DAM is intended for companies purchasing and selling electricity in an active and safe manner with ongoing implementation at specific times of the day (see: Mielczarski, 2000, and TGE S.A., 2020).

Under DAM, hourly and block contracts (base, peak and off-peak) are available for electricity transactions, with changes currently being interpreted by six price indices relating to the day and time of day of delivery (see: TGE S.A., 2020).

This work is an attempt to effectively continue the previously published authors' investigations, concerning parametric identification and modeling in the space

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[3] the arx model obtained as a result of identification includes these three parameters: [θ]=[*na nb nk*], e.g. arx431, where, in the particular case of this analysis: $na = 4$, $nb = 3$, $nk = 1$.

of transaction statuses on the DAM PEPE, and, in particular, of the following series of studies:

-   state space model and implementation for the Polish Power Exchange in MATLAB and Simulink environment (see: Marlęga, 2017),

-   identification of the Polish Power Exchange based on the data related to the Day-Ahead Market (see: Tchórzewski and Marlega, 2016),

-   modelling and simulation in the control- and systems- inspired representation of the Polish Electricity Exchange (see: Tchórzewski and Marlęga, 2017),

-   interpretation of state variables of the control model of the Polish Power Exchange System based on the Day-Ahead Market data (see: Tchórzewski and Marlęga, 2020),

in which it was shown, among others, that in order to identify the PEPE system, we downloaded from the website of Towarowa Giełda Energii S.A. (see: TGE S.A., 2020) the figures for the DAM from the time period extending from January 1, 2013, to April 30, 2016, thus creating appropriately long contractual measurement time series.

Thus, 24 input quantities were distinguished regarding the total volume of ee delivered and sold from all transactions at the trading session for a given hour of the day at particular hours of the day [in MWh] and individual output quantities related to the obtained average weighted volume price of ee from all transactions at the trading session for a given hour of the day [in PLN/MWh], obtaining thereby the basis for creating the Multi Input Singe Output (MISO) models (see: Gan, 2016; Mielczarski, 2000; Kaczorek et al., 2014; Paska, 2007).

Then, identification modeling was carried out, leading to obtaining of discrete parametric models of the DAM system for each hour of the day, assuming appropriately long measurement periods, with one day progression or with one hour progression, for all of the input quantities and for the individual subsequent output quantities.

These models were converted into continuous parametric models, and those into continuous models in the state space using the MATLAB and Simulink environments (see: Horla and Królikowski, 2010; Marlęga, 2017; Gan, 2016; Tchórzewski, 2013, 2016; Tchórzewski and Chyży, 2009; Tchórzewski and Marlęga, 2016, 2017, 2019, 2020; Park et al., 2009; Qin et al., 2009; TGE S.A., 2020; Guide for MATLAB, 2019), followed by the analysis of the results obtained and an attempt to interpret them in terms of control theory and systems theory on the one hand and development engineering on the other (see: Malko, 2013; Kwaterand and Krutys, 2016; Mielczarski, 2000; Kaczorek et al., 2014; Tchórzewski, 1993, 2013; Mynarski, 1993; Weron and Weron, 2000; Paoletti et al., 2020; Ram et al., 2010; Sarkka and Hartikainen, 2010; Tchórzewski and Marlega, 2016).

3. System, model, metamodel

The system subject to identification and metaidentification is the Day-Ahead Market System, operating on the Polish Power Exchange (Mielczarski, 2000). Identification of the DAM system leads to obtaining the catalogue of DAM system models, and identification of the parameters of the DAM system model leads to obtaining of the metamodels of the DAM system.

Thus, the results of the studies on the metaidentification of the Polish Power Exchange system were obtained on the basis of the outcomes from the identification carried out using data listed on the Day-Ahead Market regarding the volume of ee delivered and sold at particular hours of the day as input quantities and the weighted average unit price of energy electricity received at specific hours of the day, which were published, among others, in, for instance, Tchórzewski and Marlęga (2016, 2020). As a result of identification, 35 parametric models of discrete linear arx MISO type (with 24 input sizes and single output sizes) were obtained for each hour of the day, i.e. the total of 840 models.

Model parameters, i.e. coefficients of polynomials $A_j(z)$ and $B_i(z)$ were then used for the second step or level of identification, called metaidentification, in which the input values were the parameters of polynomials related to the input quantities $B_i(z)$, meaning, as mentioned, the volume of electricity supplied and sold in particular hours of the day, and the output quantities were the parameters of the polynomial associated with output quantities, i.e. with average unit prices weighted by the volume of electricity supplied and sold on the DAM in each hour of the day.

As a result of the metaidentification, carried out in this way, a metamodel of the DAM system was obtained, just as the DAM system models were obtained as a result of identification of the DAM system. It was assumed that the DAM management system model is the replacement scheme of the DAM management system, obtained as a result of identification of the DAM management system, as a parametric model, based on the ee volume input quantities recorded on the DAM in each hour of the day and the volume weighted average price output types of ee.

Further, in line with the above mentioned models, considered in Tchórzewski and Marlęga (2020), assume, in particular, that the metamodel of the DAM management system is a substitute scheme of the DAM management system model obtained as a result of metaidentification of the DAM management system, as a parametric metamodel obtained in the results of the metaidentification process using the values of parameters of the catalogue of parametric models with input values being the type of polynomial coefficients $B_i(z)$ for each hour of the day and the output quantities of the type of polynomial coefficients $A_j(z)$ for each hour of the day in the rolling model.

By further transforming the parametric model into a model in the state space, the following matrices are obtained: **A** - system (process) matrix, i.e. the degree of internal organization of the DAM system, **B** - input matrix, i.e. the control level of the

DAM system, **C** - output matrix, i.e. measurement system responses, **D** - transmission matrix of the DAM system:

$$
A(p1) = \begin{bmatrix}
0 & 0 & 0 & 0 & -0.5540 \\
0.5000 & 0 & 0 & 0 & -1.7521 \\
0 & 1.0000 & 0 & 0 & -1.7521 \\
0 & 0 & 4.0000 & 0 & -3.6676 \\
0 & 0 & 0 & 4.0000 & -2.8606
\end{bmatrix},
$$

$$
\mathbf{A(p1)} = \begin{bmatrix}
0 & 0 & 0 & 0 & -0,5540 \\
0,5000 & 0 & 0 & 0 & -1,7521 \\
0 & 1,0000 & 0 & 0 & -1,7521 \\
0 & 0 & 4,0000 & 0 & -3,6676 \\
0 & 0 & 0 & 4,0000 & -2,8606
\end{bmatrix}
$$

$$
\mathbf{B(p1)} = \begin{bmatrix}
0,0119 & 0,0259 & -0,0480 & 0,0134 & -0,0146 & 0,0039 & -0,0001 & 0,0091 & 0,0133 & 0,0108 & -0,0218 & -0,0159 \\
0,0108 & -0,0172 & 0,0139 & 0,0020 & -0,0099 & 0,0103 & -0,0037 & -0,0021 & -0,0122 & 0,0089 & 0,0056 & -0,0012 \\
-0,0043 & 0,0071 & -0,0149 & 0,0079 & 0,0010 & 0,0001 & 0,0017 & 0,0016 & 0,0021 & 0,0079 & -0,0134 & -0,0002 \ldots \\
0,0036 & -0,0073 & 0,0136 & -0,0055 & 0,0003 & -0,0012 & -0,0007 & -0,0045 & -0,0019 & 0,0010 & 0,0016 & 0,0063 \\
-0,0106 & 0,0141 & -0,0316 & 0,0177 & 0,0020 & 0,0019 & 0,0034 & 0,0053 & 0,0026 & 0,0139 & -0,236 & -0,0037 \\[4pt]
0,0266 & -0,0049 & -0,0176 & 0,0219 & -0,0083 & 0,0068 & 0,0196 & -0,0184 & -0,0043 & 0,0041 & 0,0139 & -0,0132 \\
-0,0099 & 0,0089 & 0,0001 & -0,0013 & -0,0094 & 0,0100 & -0,0031 & -0,0051 & 0,0128 & -0,0042 & 0,0077 & -0,0015 \\
\ldots \; 0,0027 & 0,0030 & 0,0005 & 0,0034 & -0,0006 & -0,0037 & 0,0050 & -0,0024 & -0,0011 & -0,0008 & 0,0028 & -0,0032 \\
-0,0118 & 0,0035 & -0,0007 & 0,0023 & -0,0048 & 0,0071 & -0,0036 & -0,0013 & 0,0083 & -0,0008 & 0,0032 & 0,0008 \\
0,0101 & 0,0045 & 0,0031 & 0,0024 & 0,0021 & -0,0118 & 0,0097 & -0,0023 & -0,0064 & -0,0018 & 0,0024 & -0,0054
\end{bmatrix}
\tag{2}
$$

$$
\mathbf{B}(p1) = \begin{bmatrix} \mathbf{B1}(p1) & \mathbf{B2}(p1) \end{bmatrix},
$$

$$
\mathbf{B1}(p1) = \begin{bmatrix}
0.0119 & 0.0259 & -0.0480 & 0.0134 & -0.0146 & 0.0039 & -0.0001 & 0.0091 & 0.0133 & 0.0108 & -0.0218 & -0.0159 \\
0.0108 & -0.0172 & 0.0139 & 0.0020 & -0.0099 & 0.0103 & -0.0037 & -0.0021 & -0.0122 & 0.0089 & 0.0056 & -0.0012 \\
-0.0043 & 0.0071 & -0.0149 & -.0079 & 0.0010 & 0.0001 & 0.0017 & 0.0016 & 0.0021 & 0.0079 & -0.0134 & -0.0002 \\
0.0036 & -0.0073 & 0.0136 & -0.0055 & 0.0003 & -0.0012 & -0.0007 & -0.0045 & -0.0019 & 0.0010 & 0.0016 & 0.0063 \\
-0.0106 & 0.0141 & -0.0316 & 0.0177 & 0.0020 & 0.0019 & 0.0034 & 0.0053 & 0.0026 & 0.0139 & -0.2360 & -0.0037
\end{bmatrix},
$$

$$
\mathbf{B2}(p1) = \begin{bmatrix}
0.0266 & -0.0049 & -0.0176 & 0.0219 & -0.0083 & 0.0068 & 0.0196 & -0.0184 & -0.0043 & 0.0041 & 0.0139 & -0.0132 \\
-0.0099 & 0.0089 & 0.0001 & -0.0013 & -0.0094 & 0.0100 & -0.0031 & -0.0051 & 0.0128 & -0.0042 & 0.0077 & -0.0015 \\
0.0027 & 0.0030 & 0.0005 & 0.0034 & -0.0006 & -0.0037 & 0.0050 & -0.0024 & -0.0011 & -0.0008 & 0.0028 & -0.0032 \\
-0.0118 & 0.0035 & -0.0007 & 0.0023 & -0.0048 & 0.0071 & -0.0036 & -0.0013 & 0.0083 & -0.0008 & 0.0032 & 0.0008 \\
0.0101 & 0.0045 & 0.0031 & 0.0024 & 0.0021 & -0.0118 & 0.0097 & -0.0023 & -0.0064 & -0.0018 & 0.0024 & -0.0054
\end{bmatrix},
$$

$$
\mathbf{C}(p1) = \begin{bmatrix} 0 & 0 & 0 & 0 & 2 \end{bmatrix},
$$

$$
\mathbf{D}(p1) = 0.
$$

Thus, there are five equations of the state and one of the output, having the following form:

$$\frac{dx_1}{dt} = -0.5540 * x_5 + 0.0119 * u_1 + 0.0259 * u_2 - 0.0480 * u_3 + 0.0134 * u_4 -$$
$$-0.0146 * u_5 + 0.0039 * u_6 - 0.0001 * u_7 + 0.0091 * u_8 + 0.0133 * u_9 + 0.0108 * u_{10} -$$
$$0.0218 * u_{11} - 0.0159 * u_{12} + 0.0266 * u_{13} - 0.0049 * u_{14} - 0.0176 * u_{15} + 0.0219 *$$
$$u_{16} - 0.0083 * u_{17} + 0.0068 * u_{18} + 0.0196 * u_{19} - 0.0184 * u_{20} - 0.0043 * u_{21} +$$
$$0,0041 * u_{22} + 0,0139 * u_{23} - 0,0132 * u_{24},$$

$$\frac{dx_2}{dt} = 0.5000 * x_1 - 1.7521 * x_5 + 0.0108 * u_1 - 0.0172 * u_2 + 0.0139 * u_3 +$$
$$+0.0020 * u_4 - 0.0099 * u_5 + 0.0103 * u_6 - 0.0037 * u_7 - 0.0021 * u_8 - 0.0122 * u_9 +$$
$$+0.0089 * u_{10} + 0.0056 * u_{11} - 0.0012 * u_{12} - 0.0099 * u_{13} + 0.0089 * u_{14} + 0.0001 *$$
$$u_{15} - 0.0013 * u_{16} - 0.0094 * u_{17} + 0.0100 * u_{18} - 0.0031 * u_{19} - 0.0051 * u_{20} +$$
$$+0.0128 * u_{21} - 0.0042 * u_{22} + 0.0077 * u_{23} - 0.0015 * u_{24},$$

$$(3)$$

$$\frac{dx_3}{dt} = 0.1000 * x_1 - 1.3873 * x_5 - 0.0043 * u_1 + 0.0071 * u_2 - 0.0149 * u_3$$
$$+0.0079 * u_4 + 0.0010 * u_5 + 0.0001 * u_6 + 0.0017 * u_7 + 0.0016 * u_8 + 0.0021 * u_9 +$$
$$+0.0079 * u_{10} - 0.0134 * u_{11} + 0.0002 * u_{12} + 0.0027 * u_{13} + 0.0030 * u_{14} + 0.0005 *$$
$$u_{15} + 0.0034 * u_{16} - 0.0006 * u_{17} - 0.0037 * u_{18} + 0.0050 * u_{19} - 0.0024 * u_{20} -$$
$$-0.0011 * u_{21} - 0.0008 * u_{22} + 0.0028 * u_{23} - 0.0032 * u_{24},$$

$$\frac{dx_4}{dt} = 4.000 * x_3 - 3.6676 * x_5 + 0.0036 * u_1 - 0.0073 * u_2 + 0.0136 * u_3 -$$
$$-0.0055 * u_4 + 0.0003 * u_5 - 0.0012 * u_6 - 0.0007 * u_7 - 0.0045 * u_8 - 0.0019 * u_9 +$$
$$+0.0010 * u_{10} + 0.0016 * u_{11} + 0.0063 * u_{12} - 0.0118 * u_{13} + 0.0035 * u_{14} - 0.0007 *$$
$$u_{15} + 0.0023 * u_{16} - 0.0048 * u_{17} + 0.0071 * u_{18} - 0.0036 * u_{19} - 0.0013 * u_{20} +$$
$$+0.0083 * u_{21} - 0.0008 * u_{22} + 0.0032 * u_{23} + 0.0008 * u_{24},$$

$$\frac{dx_5}{dt} = 4.000 * x_4 - 2.8606 * x_5 - 0.0106 * u_1 + 0.0141 * u_2 - 0.0316 * u_3 +$$
$$+0.0177 * u_4 + 0.0020 * u_5 + 0.0019 * u_6 + 0.0034 * u_7 + 0.0053 * u_8 + 0.0026 * u_9 +$$
$$+0.0139 * u_{10} - 0.0236 * u_{11} - 0.0037 * u_{12} + 0.0101 * u_{13} + 0.0045 * u_{14} + 0.0031 *$$
$$u_{15} + 0.0024 * u_{16} + 0.0021 * u_{17} - 0.0118 * u_{18} + 0.0097 * u_{19} - 0.0023 * u_{20} -$$
$$-0.0064 * u_{21} - 0.0018 * u_{22} + 0.0024 * u_{23} - 0.0054 * u_{24},$$

and

$$y_1(t) = 2.0000 \, x_5(t). \qquad (4)$$

The previously referred to work of Tchórzewski and Marlęga (2020) also shows how to interpret the obtained state variables of the PEPE models for particular hours of the day and selected elements of matrices **A**, **B** and **C,** without the matrix **D**, which was a matrix of zero elements. The interpretation of state variables was started from the equation of the output of the DAM state variable model (see Marlęga, 2016, Tchórzewski and Marlęga, 2016, 2020), in which the output variable $y_1(t)$ was the volume-weighted average price obtained from the supply and sale of electricity in hours 0-1 in contractual 184 days resulting from the model identification period [expressed in PLN/MWh].

Then, assuming that the $c_{11}=2,0000$ element of matrix **C** is expressed in PLN/MWh^2, then the state variable $x_5(t)$ was interpreted as electricity supplied and sold on PEPE in the hour 0-1 during the quotation period on the DAM [in MWh], i.e. in the period from 01/01/2013 to 30/06/2013. On the basis of the state variable $x_5(t)$ and using the state equation to determine the state variable $x_1(t)$, the state variable $x_1(t)$ was interpreted as the power resulting from the ee delivered and sold in hours 0-1, interpreting element a_{11} as the frequency of its changes on the day of sale [1/day].

Further, based on the state variable $x_1(t)$ and using the state equation for determining the state variable $x_2(t)$ and the state equation for determining the state variable $x_3(t)$, they were interpreted as state variables expressing electricity sold during the day, respectively.

Finally, the interpretation of the state variable $x_4(t)$ results from the interpretation of the state variable $x_3(t)$ and the state equation for determining the derivative of the state variable $x_4(t)$, which was interpreted as electricity supplied to the stock exchange and sold during the measurement period, i.e. in within 184 days, namely from 01/01/2013 to 30/06/2013 (see: Marlęga, 2016, Tchórzewski and Marlęga, 2017, 2020).

4. Metamodels of the Day-Ahead System in the approach of control theory and systems engineering

4.1. Metaidentification as identification of DAM models

System identification is a department of mathematical modeling of systems based on experimental data, i.e. a way of modeling dynamic systems, in which the model is created using data from observation and measurement. In the case of system engineering, the dynamic system may have the schematic form shown (see: Tchórzewski, 1992, Mynarski [ed.], 1993, Weron and Weron, 2000) as shown here in Fig. 1., in which one can speak about the system's inputs and outputs. Depending on the number of entries and exits, the following four types of models: SISO, SIMO, MISO and MIMO are distinguished.

Thus, the Day-Ahead Market system in a systemic perspective can be presented as a dynamic system, which has the form shown (see: Marlęga, 2016; Tchórzewski and Marlęga, 2016, 2017, 2019, 2020) in Fig. 2., where:

S_i(DAM) – is the i-th subsystem of the Energy Market, conceived as the Towarowa Giełda Energii S.A. system,

S_{i+1} – is the (i+1) - st subsystem of integrated electricity reception from TGE S.A. as an operating system for the TGE S.A. system,

S_{i-1} – is the (i-1) - st subsystem of the integrated electricity supplier (power plant) at TGE S.A. as a security system for the TGE S.A. system,

SZ_i(DAM) – is the management system in the TGE S.A. system as in the i-th REE subsystem,

SW_i(DAM) – is the executive system in the TGE S.A. system as in the *i*-th REE subsystem,

PK_i(SZ,FD-FI) – is the management subsystem in the *i*-th DAM TGE S.A. system,

PW_i(SZ,IF-DF) – is the executive subsystem in the Management System in the *i*-th DAM TGE system,

PI(PZ,IF-FI) – the information subsystem in the Management System in the *i*-th DAM TGE system,

PD(PZ,FD-DF) – is the decision subsystem in the Management System in the *i*-th DAM TGE system,

FD(TGE,*t*) – is the income stream of the utility in the *i*-th DAM TGE system at time *t*,

FI(DAM,*θ*,*t*) – is the expense stream of the utility in the *i*-th DAM system in the long time period *θ* regarding the current time *t*,

IF(DAM,*θ*,*t*) – is information stream in the *i*-th DAM subsystem as the TGE system at time *θ* and at time *t*, being the information on the volume of electricity delivered and sold to consumers (DAM, ee),

DF(DAM,*θ*,*t*) – is the decision stream in the *i*-th DAM subsystem as the TGE system at time *θ* and at time *t*, being the decision on the average price per MW weighted by the volume of electricity supplied and sold to consumers C (DAM,*θ*,*t*),

ZI(DAM,*θ*,*t*) – is the flow of electricity supplied for sale in the *i*-th subsystem of the DAM system as the TGE S.A. subsystem at time *θ* and at time *t*,

ZD(DAM,*θ*,*t*) – is the stream of electricity sold to consumers in the *i*-th DAM subsystem as the TGE S.A. system at time *θ* and at time *t*.

The Day-Ahead Market System is secured (it receives information about possible electricity supplies) from the side of the power plant system and other sources, including renewable sources for electricity, so in this dissertation it is assumed in the form of the volume of electricity supplied and sold on the Day-Ahead Market. As a result of the operation of the exchange on the DAM, electricity is sold to customers according to the weighted average price by the delivered volume. Fig. 2. shows three types of models, i.e. the praxiological model (MP), the cybernetic model (MC) and the system model (MS) (Kaczorek et al., 2014; Konieczny, 1983; Tchórzewski, 1990, 1992, 2013).

The praxiological model is composed of three types of systems, i.e. the basic system, which is the DAM system, the aggregated system, which in this case consists of systems supplying electricity, and the aggregate system, which consists of the systems of electricity consumers.

Figure 2. Selected types of Day-Ahead Market system models on Towarowa Giełda Energii S.A. in terms of control theory and systems engineering. Designations: MP (DAM) - DAM TGE praxeological model, MC - DAM TGE cybernetic model, MS (SZ) - TGE Day-Ahead Management system model, OTO (FI-FD) - system environment of the DAM Management System at TGE, other markings in text.
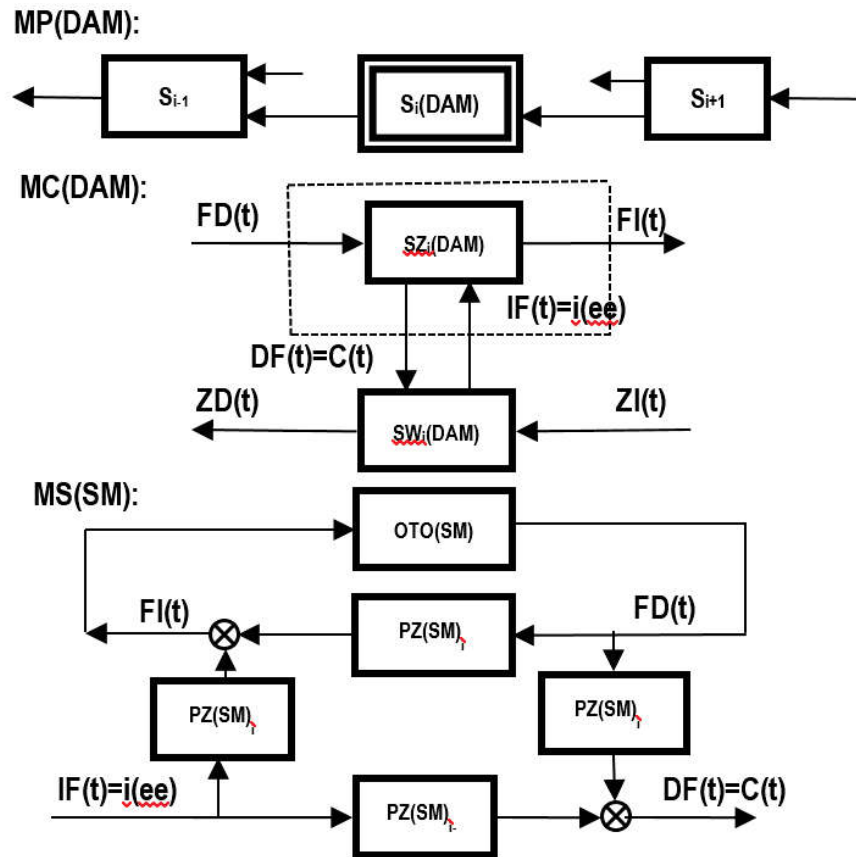
Source: own study based on the works (see: Kaczorek at all, 2014; Konieczny, 1983, Tchórzewski, 1990, 1992, 2013).

The cybernetic model is created as a result of transverse subdivision into separate submodels (subsystems) of the praxiological model, that is, inter alia, of the DAM system into two subsystems: the management system (SZ) and the executive (implementation) system (SW).

On the other hand, **the system model** is created as a result of the division of the cybernetic model into subsystems due to the generated outputs, hence the result of such division is a model composed of four basic subsystems, that is:

PZ(FD-FI) - subsystem in the management system responsible for making managerial decisions, including general financial decisions,

PZ(IF-FI) - subsystem in the management system responsible for generating information, including, inter alia, financing the purchase of supplied and sold electricity,

PZ(FD-DF) - subsystem in the management system responsible for making decisions regarding the purchase of the volume of electricity,

PZ(IF-DF) - subsystem in the management system responsible for informing the management about the manner of executing decisions concerning the purchase of electricity.

Therefore, numerical data were adopted for identification, as already indicated, from the period between January 1st, 2013, and April 30th, 2016, and with their use identification was carried out on six-month samples (184 days) with the progress of one month, thereby obtaining the total of 35 parametric models of discrete linear arx and, consequently, 35 continuous models, as well as models in the state space (see: Kaczorek et al., 2014; Marlęga, 2016; Tchórzewski, 2013). An example of the input and output data waveform is shown in Fig. 3.

Figure 3. Model of DAM system coefficient changes for 35 contractual measurement periods. Designations: u_1 - input from the metamodel for the z^{-1} delay factor occurring in the $B_1(z)$ polynomial of the DAM model, y_1 - output from the metamodel for the z^{-1} delay factor occurring in the $A_1(z)$ polynomial of the DAM model.
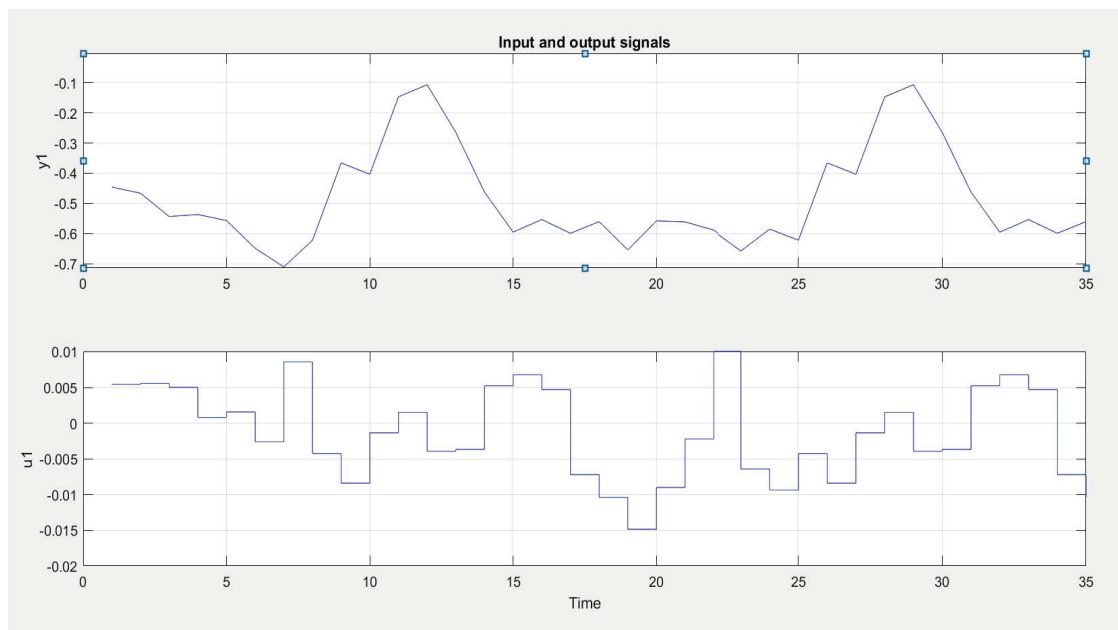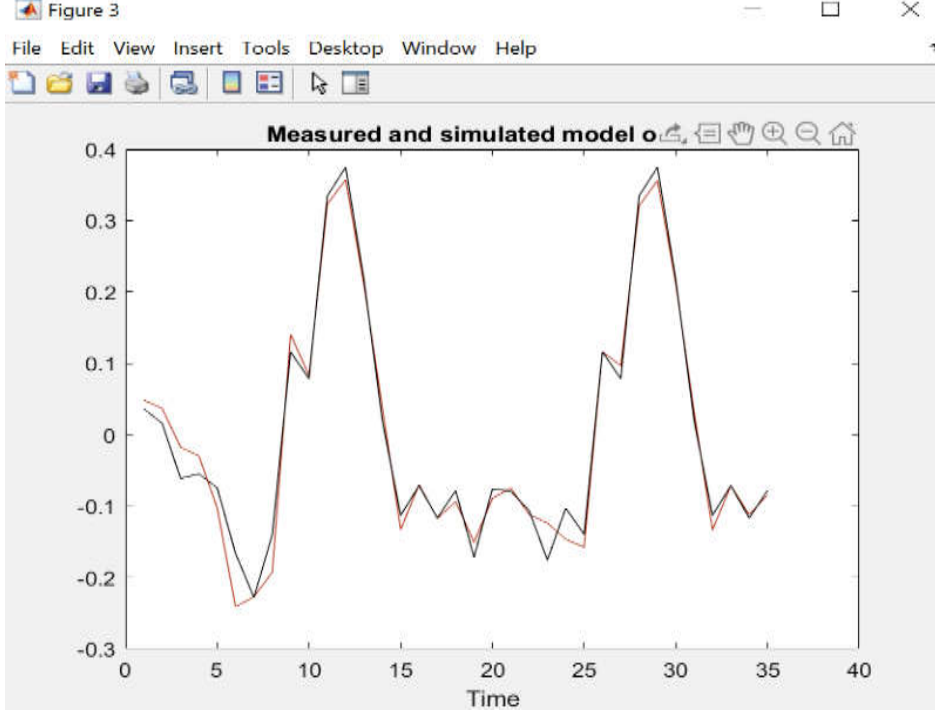

Source: Own elaboration in MATLAB and Simulink using the System Identification Toolbox (see: TGE S.A., 2020; Guide for MATLAB, 2019).

4.2. Parametric metamodel of the DAM

The metaidentification, the results of which are provided in the present work, was carried out using four coefficients of $B_1(z)$ polynomials as input quantities and one coefficient of $A_1(z)$ polynomials as the output quantity, with the respective values given in Table 1. As a result of metaidentification, a discrete parametric arx linear model was obtained (see Fig. 4. further on).

Figure 4. Example of metaidentification results, in which the DAM system metamodel was obtained. Designations: *x* axis (time) - long identification time [half-year], *y* axis - coefficient standing at the delay of z^{-1} in $A_1(z)$.

Source: own elaboration in MATLAB and Simulink enviroment using the System Identification Toolbox (see: TGE S.A., 2020; Guide for MATLAB, 2019).

The here considered arx discrete metamodel (mm), where *na*=14, *nb*=4, *nk*=1, of the DAM system of *p*1 period (marked as: arxmmDAMp1z1441) has the following form

$$A_1(z)\, y(t) = B_1(z)\, u_1(t) + B_2(z)\, u_2(t) + B_3(z)\, u_3(t) + B_4(z)\, u_4(t) + e(t), \qquad (5)$$

where:

$$A_1(z) = 1 - 0.6313 \cdot z^{-1} + 0.07469 \cdot z^{-2} + 1.193 \cdot z^{-3} + 0.7858 \cdot z^{-4},$$

$$B_1(z) = 4.65 \cdot z^{-1} - 2.772 \cdot z^{-2} - 14.1 \cdot z^{-3} + 4.542 \cdot z^{-4},$$

$$B_2(z) = 14.51 \cdot z^{-1} - 3.044 \cdot z^{-2} + 1.087 \cdot z^{-3} + 10.75 \cdot z^{-4},$$

$$B_3(z) = -22.7 \cdot z^{-1} - 25.97 \cdot z^{-2} - 5.716 \cdot z^{-3} + 4.925 \cdot z^{-4},$$

$$B_4(z) = -5.128 \cdot z^{-1} + 0.2273 \cdot z^{-2} + 6.893 \cdot z^{-3} - 1.938 \cdot z^{-4}.$$

Then, the discrete parametric model (5) was converted to a continuous parametric model to obtain the: arx continuous metamodel of the DAM system of *p*1 period (marked as: thmmDAM):

$$A_1(s)\, y_1(t) = B_1(s)\, u_1(t) + B_1(s)\, u_1(t) + B_2(s)\, u_2(t) + B_3(s)\, u_3(t) + B_4(s)\, u_4(t) + C(s)\, e(t), \quad (6)$$

where:

$$A_1(s) = s^4 + 0.2411\, s^3 + 7.313\, s^2 - 3.23\, s + 5.171,$$

$B_1(s) = -11.95\ s^3 + 25.61\ s^2 + 7.375\ s - 16.4,$

$B_2(s) = -1.314\ s^3 + 49.3\ s^2 + 4.693\ s + 49.75,$

$B_3(s) = -14.68\ s^3 - 16.16\ s^2 - 111.3\ s - 105.6,$

$B_4(s) = 3.755\ s^3 - 16\ s^2 - 15.41\ s + 0.117,$

$C(s) = s^4 + 1.06\ s^3 + 6.837\ s^2 + 2.997\ s + 2.135.$

Table 1. Example of $B_i(z)$ and $A_1(z)$ polynomials' coefficients obtained as a result of identification of 35 models of the DAM system for hour 0-1.

period	$B_1(z^{-1})$	$B_2(z^{-2})$	$B_3(z^{-3})$	$B_4(z^{-4})$	$A_1(z^{-1})$
p1	0.005455	0.01089	0.0006017	0.006013	-0.4456
p2	0.005572	0.001121	0.0034	0.002929	-0.4664
p3	0.005025	-0.004956	0.001012	0.001906	-0.5432
p4	0.000816	-0.0006004	-0.0000086	0.007244	-0.5367
p5	0.001582	- 0.006111	0.001007	0.005847	- 0.5567
p6	-0.002599	- 0.001136	0.002666	0.004478	- 0.6487
p7	0.008594	- 0.006689	0.001814	0.005702	- 0.7101
p8	-0.004250	0.006656	0.004926	0.001608	- 0.6217
p9	-0.008400	- 0.000507	0.005878	0.005528	- 0.3658
p10	-0.001345	0.004507	0.002137	0.001677	- 0.4035
p11	0.001526	0.0024	- 0.001671	0.003377	- 0.1471
p12	-0.003933	0.007045	- 0.00472	- 0.00268	- 0.1071
p13	-0.003672	0.008857	- 0.005701	0.00306	- 0.265
p14	0.005230	0.009797	- 0.007371	- 0.00031	- 0.4625
p15	0.006782	0.0108	- 0.005087	0.007766	- 0.595
p16	0.004715	0.01171	0.004841	0.008003	- 0.5533
p17	-0.007193	0.008944	0.006313	0.001716	- 0.5988
p18	-0.010380	0.005976	0.009154	0.002155	- 0.5602
p19	-0.014860	- 0.001821	0.002023	0.01029	- 0.6538
p20	-0.009000	- 0.004485	0.01131	0.01088	- 0.5578
p21	-0.002215	- 0.00174	0.01208	0.01367	- 0.5611
p22	0.010050	- 0.003868	- 0.00033	0.000134	- 0.5883
p23	-0.006413	0.0001901	0.007854	0.001096	- 0.6574
p24	-0.009362	0.007667	0.001022	- 0.00623	- 0.5849
p25	-0.00425	0.006656	0.004926	0.001608	- 0.6217
p26	-0.0084	- 0.000508	0.005878	0.005528	- 0.3658
p27	-0.001345	0.004507	0.002137	0.001677	- 0.4035
p28	0.001526	0.0024	- 0.00167	0.003377	- 0.1471
p29	-0.003933	0.007045	- 0.00472	- 0.00268	- 0.1071
p30	-0.003672	0.008857	- 0.005701	0.00306	- 0.265
p31	0.00523	0.009797	- 0.007371	- 0.00031	- 0.4625
p32	0.006782	0.0108	- 0.005087	0.007766	- 0.595
p33	0.004715	0.01171	0.004841	0.008003	- 0.5533
p34	-0.007193	0.008944	0.006313	0.001716	- 0.5988
p35	-0.01038	0.005976	0.009154	0.002155	- 0.56020

Source: Own elaboration in MATLAB and Simulink environment using the System Identification Toolbox (see: TGE S.A., 2020; Guide for MATLAB, 2019)

4.3. Metamodel in the state space of the DAM system

The continuous parametric model (6) was thereafter converted into a continuous model in the state space using the th2ss() function, to obtain the matrices:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & -1.2928 \\ 1 & 0 & 0 & 0.8074 \\ 0 & 1.0000 & 0 & -1.8284 \\ 0 & 0 & 4.0000 & -0.2411 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} -0.5126 & 1.5547 & -3.3000 & 0.0037 \\ 0.2305 & 0.1467 & -3.4779 & -0.4816 \\ 0.8003 & 1.5406 & -0.5049 & -0.5000 \\ -1.4942 & -0.1643 & -1.8347 & 0.4694 \end{bmatrix}, \qquad (7)$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 8.0000 \end{bmatrix},$$

$$\mathbf{D} = [0\ 0\ 0\ 0],$$

$$\mathbf{K} = \begin{bmatrix} -0.0949 \\ 0.1946 \\ -0.0149 \\ 0.1024 \end{bmatrix}.$$

So, the following metamodel in the state space was obtained, i.e. the state equation:

$$\dot{x}_{1m} = -1.2929x_{1m}(t) - 0.5126u_1(t) + 1.5547u_2(t) - 3.3u_3(t) + 0{,}0037u_4(t),$$

$$\dot{x}_{2m}(t) = x_{1m}(t) + 0.8074x_{4m}(t) + 0.2305u_1(t) + 0.1467u_2(t) - 3.4779u_3(t) - 0.4816u_4(t),$$

$$\dot{x}_{3m}(t) = x_{2m}(t) - 1.8284x_{4m} + 0.8003u_1(t) + 1.5406u_2(t) - 0.5049u_3(t) - 0.5u_4(t), \qquad (8)$$

$$\dot{x}_{4m}(t) = 4.0x_{3m}(t) - 0.2411x_{4m}(t) - 1.4942u_1(t) - 0.1643u_2(t) - 1.8347u_3(t) + 0.4694u_4(t),$$

and the output equation:

$$y_{1m}(t) = 8.0 \cdot x_{4m}(t), \qquad (9)$$

where:

index m - indication that the variables relate to a metamodel,

x_{im} – i-th metamodel state variable, $i = 1,\ldots,4$.

It can be seen that just as it was possible to interpret the state variables of the DAM models, so also can the state variables of the DAM metamodel be interpreted. But it is worth noting that in this case the inputs to the metamodel are the $B_i(z)$ polynomial coefficients of the DAM models, and the outputs are the coefficients of the polynomials $A_1(z)$.

Research related to the interpretation of the metamodel state variables must also be preceded by the research on the interpretation of the coefficients of the RDN system models, which will be considered in the authors' subsequent publications. It can only be noted that the interpretation should be started from the state variable x_4, which can be interpreted on the basis of the output equation (9), assuming a specific interpretation of the element c_{14} of the matrix **C**.

It can be further noticed that the output from the metamodel has the nature of the coefficient a_{11}, occurring in the polynomial $A_1(z)$ of the DAM system model, which expresses the ratio of the volume weighted average price of electricity at time t to the price at time t-1 (in the case under consideration in the previous hour).

Research on the interpretation of the meta-model state variables, as well as on the interpretation of the functioning of the control system, which is the DAM meta-model, is continued.

5. Conclusions and directions of further research

It is possible to carry out meta-identification of the Polish Electric Power Exchange system using numerical data, e.g. on the Day-Ahead Market.

With that aim 35 catalogues of DAM models were generated, whose parameters were used to carry out secondary identification, that is, the metaidentification, in which DAM models were subject to identification, not the DAM systems.

It was also shown that parametric models as well as variable state metamodel of the models of the DAM system can be obtained and then also basing on them a search for models of DAM systems could be performed.

The research can be continued, especially in the field of interpretation of the obtained variables of the DAM metamodel.

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METAIDENTIFICATION OF THE DAY-AHEAD MARKET SYSTEM OF THE POLISH ELECTRIC POWER EXCHANGE IN TERMS OF CONTROL AND SYSTEMS THEORY

Abstract: This paper contains the results of investigations concerning the metaidentification of the Day Ahead Market System as a Management System, which were obtained on the basis of a catalogue of 35 models of the Day Ahead Market System. In metaidentification, the input quantities are parameters of polynomials associated with the input quantities of the Day Ahead Market System, i.e. the volume of electricity supplied and sold at particular times of the day, and the output quantities are the parameters of the polynomial associated with the output quantities, i.e. with the weighted average electricity prices obtained in the result of the sale of electricity every hour of the day. As a result of metaidentification, a metamodel of the Day Ahead Market System is obtained, just like the identification results in Day Ahead Market System models. Discrete parametric models, parametric continuous models and continuous models in the state space were obtained.

Keywords: modelling, identification, multi-modelling, polynomials, meta-identification, Day Ahead Market

METAIDENTYFIKACJA SYSTEMU POLSKIEJ GIEŁDY ENERGII ELEKTRYCZNEJ TGE24 W UJĘCIU TEORII STEROWANIA I SYSTEMÓW

Streszczenie: Artykuł zawiera wyniki badań, dotyczących metaidentyfikacji systemu giełdy energii elektrycznej TGE24 jako systemu zarządzania, uzyskanych na podstawie katalogu 35 modeli tego systemu. W tym badaniu, wielkości wejściowe to współczynniki wielomianów, odnoszących się do zależności wejścia-wyjścia badanego systemu, tj. wielkości energii dostarczonej i sprzedanej w określonych momentach, zaś wielkości wyjściowe to współczynniki, związane z cenami energii elektrycznej, zrealizowanymi w poszczególnych momentach. W wyniku metaidentyfikacji otrzymano metamodel badanego układu. Otrzymano modele parametryczne dyskretne i ciągłe oraz modele ciągłe w przestrzeni stanów.

Słowa kluczowe: modelowanie, identyfikacja, wielomodelowość, wielomiany, meta-identyfikacja, rynek energii elektrycznej